ACKNOWLEDGEMENT

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COMMENTS ON HEAT TRANSFER BETWEEN SOLID PARTICLES AND A GAS IN A NON-UNIFORMLY AGGREGATED FLUIDIZED BED

S. S. ZABRODSKY, *Int. J. Heat Mass Transfer*, **6**, 23 (1963)

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MANY workers have investigated heat transfer from gas to particles in a fluidized bed and the best survey of this field is by Frantz [1]. The experimental technique of some investigators or their interpretation of data can be questioned but there is a residual core of results that indicate Nusselt numbers less than 2. Starting from this point, Dr. Zabrodsky argues that in a bed of particles the minimum Nusselt number should be appreciably greater than 2 but he goes on to show that, in spite of this, apparent values can approach zero if one assumes certain hydrodynamic conditions within the bed. In this note it is shown first that Dr. Zabrodsky's argument about minimum Nusselt numbers can be extended and generalized but does not accord with experimental facts and second, that the hydrodynamic conditions within a gas fluidized bed are different from those assumed and are not so amenable to generalized calculation as has been supposed.

If a hot sphere of diameter d is surrounded by a concentric spherical shell of diameter d_1 which acts as a heat sink and the intervening spherical annulus is filled with stagnant fluid, Zabrodsky shows that the Nusselt number is $2/(1 - d/d_1)$. It takes its familiar limiting value of 2 therefore only when the concentric sink is infinitely distant. In a bed of packed uniform spheres, the intervening fluid can be imagined to be re-disposed as spherical shells around each particle and in this way a value can be ascribed to d_1 in order to estimate a limiting Nusselt number, that is to say, the Nusselt number with stagnant fluid ($Re = 0$). Zabrodsky writes the volume of this equivalent spherical shell as $\pi d^2\delta$ which should really be $(\pi/6)[(d + 2\delta)^3 - d^3]$ and for the case of cubic packing this leads to $Nu_{\min} = 10{\cdot}3$ instead of the value $8{\cdot}6$ given in the paper. The value is, of course, different again if one considers any other packing geometry. Morris [2] has made an identical analysis for the case of mass transfer and the limiting Sherwood number.

Consider any arrangement of equal sized spheres dispersed more or less uniformly in a fluid and let the porosity or voidage be ϵ. The amount of fluid per sphere will be $(\pi d^3/6)[\epsilon/(1 - \epsilon)]$ and, following Zabrodsky's method, this can be expressed as an equivalent spherical shell of fluid so that,

$$(\pi d^3/6)[\epsilon/(1 - \epsilon)] = (\pi/6)[(d + 2\delta)^3 - d^3]$$

whence,

$$2\delta = \left[\left(\frac{1}{1 - \epsilon}\right)^{\frac{1}{3}} - 1\right] d \qquad (1)$$

and the minimum Nusselt number becomes,

$$Nu_{\min} = 2/[1 - (1 - \epsilon)^{\frac{1}{3}}]. \qquad (2)$$

The expression is plotted in Fig. 1 and it is seen that $Nu_{\min}$ can apparently be as high as 21 for close packed spheres.

Also in Fig. 1 are shown experimental data for $\frac{1}{2}$ in diameter spheres arranged in a rhombohedral lattice. In the experiment, the spacing between spheres could be

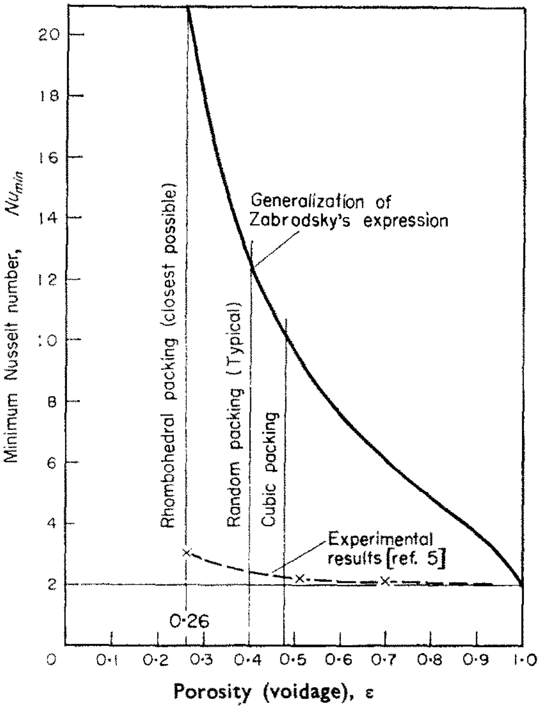

FIG. 1. Minimum Nusselt number for uniform spheres in different packings.

changed in order to vary the porosity of the whole assembly whilst preserving the geometry. The apparatus was as previously used for drag measurements [3, 4] and heat-transfer coefficients were measured by heating a single copper sphere at a known rate and recording the surface and water temperatures [5]. Minimum Nusselt numbers were obtained by extrapolating to zero heat flux and zero fluid velocity. Similar results have since been obtained with the same and with larger spheres in air. The minimum Nusselt number with spheres touching in rhombohedral packing is little more than 3, an order of magnitude less than predicted. Similarly, Mullin and Treleaven [6] show increases of Sherwood number at low Reynolds numbers of about two- or three-fold between closely packed and isolated spheres. The minimum Sherwood number seems to be perhaps twice the corresponding Nusselt number and as yet there is no explanation of this.

This very wide discrepancy between theory and apparently unambiguous experimental results suggests an incorrect assumption and this is most probably the supposition of radial symmetry. Plainly no assembly of spheres of finite size can be spherically symmetrical about a point and therefore the heat flow from any given sphere cannot be radially uniform. This is particularly so when spheres are close together. To illustrate the importance of this geometrical assumption, consider radial heat flow from an infinitely long cylinder. The expression for the limiting Nusselt number corresponding to Zabrodsky's equation (2) is $2/\log_e (d_1/d)$ which becomes zero with infinitely distant boundaries. That is to say a steady finite heat flow is impossible with a finite temperature difference. The Nusselt number relationship corresponding to equation (2) above is,

$$Nu_{\min}\,(\text{cylinder}) = 4/\log_e\,[1/(1-\epsilon)] \qquad (3)$$

which gives values appreciably less than for spheres with the same porosity. Rigorous analysis of heat flow in an assembly of spheres would be enormously complex but there is no reason to suppose that the limiting Nusselt number will not be less than 2.

Consider now Zabrodsky's treatment of particles in a fluidized bed. The heat-transfer coefficient is taken from Wadsworth's data ([12] in the paper) which applies to Reynolds number of order 10^4 but this is combined with Leva's hydrodynamic equation that Zabrodsky has extrapolated to apply up to $Re = 200$. (Leva himself sets an upper limit of $Re = 5$ and even so, the scatter range of data is a factor of 2.) Flow in Wadsworth's case is plainly turbulent whilst Leva's expression refers to streamline flow and their combination is thus invalid. Equation (9) in the paper and all that follows from it cannot therefore be accepted.

The theory of "micro breaks" might conceivably apply to a turbulent liquid fluidized bed but it is not relevant to the highly ordered pattern of gas and solids flow that occurs in a normal bubbling gas fluidized bed [7, 8, 9]. The idea of unstable particle aggregates that disappear and re-form presupposes random motion within the bed whereas, although the appearance and duration of bubbles may be partly random, solids and gas motion are uniquely determined by the bubbles.

Since the history of contacting between gas and particles can be predicted, it follows that the overall heat-transfer rate can be estimated once the relationship between Nusselt number and local Reynolds number is known. It is first necessary to calculate the rate of bubbling in the fluidized bed for this determines the rate of particle mixing, and therefore the relative positions of particles with respect to hot gas entering the bed. The next step is to calculate how gas will flow through the bed. For example, with particles less than about 0·3 mm diameter, the bubble velocity is greater than the interstitial gas velocity and a gas cloud develops around the bubble. Gas that passes through the bed in this way has a low residence time and is only partly in contact with particles in a shell around the bubble so that heat transfer may be restricted. Interstitial gas on the other hand will be in continuous contact with particles and roughly speaking, it will cool with bed height according to an exponential decay law. In this way the overall heat-transfer rate can be calculated once the hydrodynamic situation is known.

The overall Nusselt number is invariably low and often orders of magnitude less than 2. The reason for this is that the maximum driving force is not available to all the particles. Alternatively, not all the particle surface area is exposed to hot gas. This, of course, is the theme underlying Zabrodsky's treatment. Since the heat capacity of gas is small compared with that of solids and fluidizable particles have a large surface area per unit volume, heat-transfer rates are always high wherever there is a modest temperature difference so that the calculated overall heat-transfer rate is not particularly sensitive to the hydrodynamic model that is chosen. However, this is not the case when mass transfer is considered, for then the rates may be low for chemical reasons and the calculation becomes critically dependent upon the assumptions made about the details of gas solids contacting. (The symbols d, δ and Nu_{min} are as defined in Zabrodsky's paper.)

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Note: Zabrodsky's reference [12] should read WADSWORTH.

A NOTE ON HEAT TRANSFER BETWEEN SPHERICAL PARTICLES AND A FLUID IN A BED

[A reply to Dr. P. N. Rowe's comments on the author's paper: Heat transfer between solid particles and a gas in a non-uniformly aggregated fluidized bed, *Int. J. Heat Mass Transfer* **6**, 23 (1963).]

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IN THE present issue of this Journal is published a comment by Dr. P. N. Rowe [1] on the present writer's previous paper [2]. This comment is undoubtedly useful to the author since it allows him to elucidate his conceptions while discussing Dr. Rowe's contribution.

The main aim of [2] was to show the principal physical grounds for the very small apparent values of the Nusselt number, namely the actual temperature driving forces caused by micro-non-uniformity of fluid distribution in a bed. For this purpose the author used a model and correlations which were rather approximate but easy to understand. Naturally, the well-known radial asymmetry of the gas "shell" around a solid particle in a bed was not considered at this first step. However, some of Dr. Rowe's remarks show that he is wrong in thinking that the formulae in [2] were suggested as final ones for design calculations. Dr. Rowe, for example, writes about the inaccuracy of Nusselt number estimation for the cubical packing. On the contrary, it would be better to say that since this model is an approximate one, it should be used to determine rough values of Nusselt numbers but not accurate to one decimal place.

Dr. Rowe unfortunately trusts without any reasons or physical grounds that such true film-heat-transfer coefficients are possible in the system of solid particles which correspond to Nusselt numbers far below two. Comparison of the approximate true Nusselt numbers estimated by the present author's model with the Nusselt numbers obtained from Dr. P. N. Rowe's experiments (Fig. 1 of [1]), provides no evidence of such possibility but makes a good proof that Dr. Rowe's methods are not valid in their essence for determining true heat-